LEONHARDI EULERI OPERA OMNIA

sub auspiciis

ACADEMIAE SCIENTIARUM NATURALIUM HELVETICAE

Edenda curaverunt

Vanja Hug, Andreas Kleinert, Martin Mattmüller, Gleb K. Mikhaïlov,
Fritz Nagel, Norbert Schappacher, Thomas Steiner

Series quarta A

COMMERCIUM EPISTOLICUM

Volumen quartum – Pars prima

Impensis

FUNDATIONIS NATIONALIS CONFOEDERATIONIS HELVETICAE

et

ACADEMIAE SCIENTIARUM NATURALIUM HELVETICAE

Venditioni exponunt

SPRINGER BASILEAE

MMXV

LEONHARDI EULERI
COMMERCIUM EPISTOLICUM

CUM CHRISTIANO GOLDBACH

PARS I

Ediderunt
Franz Lemmermeyer et Martin Mattmüller

CORRESPONDENCE OF LEONHARD EULER

WITH CHRISTIAN GOLDBACH

PART I

edited by
Franz Lemmermeyer and Martin Mattmüller

SPRINGER BASEL

2015

$$4 = \begin{cases} 1+1+1+1 \\ 1+1+2 \\ 1+3 \end{cases} \quad 5 = \begin{cases} 2+3 \\ 1+1+3 \\ 1+1+1+2 \\ 1+1+1+1+1 \end{cases} \quad 6 = \begin{cases} 1+5 \\ 1+2+3 \\ 1+1+1+3 \\ 1+1+1+1+2 \\ 1+1+1+1+1+1 \end{cases} \&c$$

$$V^{2n+1} = (2v+1)(v+1)^{n-1}$$

$$\text{summa feriei } \frac{x^n}{n \cdot 2^{2n}}$$

$$\text{applicata} = \frac{x}{1 \cdot 2^2} + \frac{x^2}{2 \cdot 2^4} + \frac{x^3}{3 \cdot 2^6} + \frac{x^4}{4 \cdot 2^8} + \&c.$$

Moscau $\frac{27}{7}$ Jun. st. n. 1742.

Goldbach's letter nº 51 to Euler, (May 27th) June 7th, 1742: reproduction of the fourth page (PFARAN, f. 136, op. 2, n. 8, fol. 44v)

The paragraph written in the left margin contains the original form of the (ternary) Goldbach Conjecture – now the Goldbach-Helfgott Theorem (see Section 2.1.3 of the Introduction, *infra* p. 47, and nº 51, note 13).

Two remarks jotted down by Euler between the lines of Goldbach's original missive, which complement or correct other statements by Goldbach (cf. nº 51, notes 14–15), can also be seen.

Editors
Franz Lemmermeyer
Jagstzell, Germany

Martin Mattmüller
Bernoulli-Euler-Zentrum
Basel, Switzerland

TABLE OF CONTENTS

[The English translation of the letters with the endnotes and the indices are contained in vol. IVA/4, Part II.]

PREFACE

Habent sua fata editiones. Already in the first project for Leonhard Euler's *Opera Omnia*, drafted in 1910, it was mentioned that some parts at least of Euler's scientific correspondence should be included in the edition. However, priority was then given to the (re)publication of Euler's printed works in three series devoted to mathematics, to mechanics and astronomy, and to general physics; the issue of editing Euler's letters and the rest of his manuscript heritage was postponed indefinitely.

The first 20th-century initiatives for publishing part of Euler's correspondence were not related to the *Opera Omnia* edition, but originated with a group of historians of science based at the Soviet Academy of Sciences in Leningrad in the 1930s. Various inventories and collections were published in the following decades, presenting to the public for the first time Euler's correspondence with such eminent scientists as Poleni, Delisle, Lomonosov, Tobias Mayer and Bonnet. The most ambitious project of this school was planned in the context of Euler's 250th anniversary as a joint venture between the Leningrad Section of the Soviet Academy of Sciences and the Academy of Sciences of the German Democratic Republic. Between 1959 and 1976, four major volumes of Euler's letters, mostly exchanged during his Berlin period with his former colleagues at Petersburg, were published by a team led by Adol'f Pavlovich Yushkevich and Eduard Winter; this project included an edition of the Euler-Goldbach correspondence that appeared in Berlin in 1965.

Spurred by these activities, the Swiss Euler Committee finally decided in 1967 to start an additional series of the *Opera Omnia*, or rather two sub-series: IVA for the correspondence and IVB for the unpublished manuscripts (the latter was later converted into a web-based digital project that is currently being planned). Since most of the originals were preserved in the Leningrad Archive of the Soviet Academy of Sciences, and since a considerable number of Euler experts lived and worked in the Soviet Union, Series IV was conceived as a shared project of the Swiss and the Soviet Academies and led by a joint Editorial Committee.

The first volume of series IVA was published in 1975; it is an inventory of all letters from and to Euler known at that time (ca. 3100 items). For each letter (identified by a *Repertorium* number), the volume lists the date and place, version (as an original, copy, excerpt, . . .), language, size, location(s) and extant publications; a short summary follows. Three volumes of correspondence were produced up to 1998: they comprise the letters exchanged with Clairaut, d'Alembert, Lagrange, Maupertuis, Frederick II, Johann I and Nicolaus I Bernoulli.

The Goldbach correspondence had not been given a high priority, since it was already available in a modern edition. Only in 2001 was the task of editing volume IVA/4, planned as a basically new edition of the Euler-Goldbach correspondence, assigned to Günther Frei and Martin Mattmüller. The former, a number theorist and historian of mathematics from Switzerland, had held a chair at Laval Univer-

sity (Québec) for most of his career; he had published extensively on the history of number theory in the 19th and 20th centuries and been a member of the Euler Committee since 1995. The latter had graduated in mathematics in Basel in 1983, worked for the Bernoulli Edition from 1987 to 2000, and joined the Euler Edition in 2001 as a paid part-time collaborator. Later he acted as archivist of the Euler Archive in Basel and secretary of the Euler Committee.

Mattmüller started his work on the Goldbach correspondence by establishing a new transcription of the letters from (photographs of) the manuscripts. In 2002 it was decided to use English as the working language for the volume and to add an English translation of the source texts with their peculiar mixture of 18th-century German and Latin, in order to make them accessible to a wider range of non-specialist readers worldwide. Early in 2004, a provisional version of the transcriptions and translations in TEX was available. The next years were mainly spent in grouping the topics addressed in the correspondence and studying its scientific and social setting; this resulted in a draft volume submitted to the Editorial Committee, which specified the distribution of the notes and introductory sections among the two editors. However, in 2008, at the moment when he had planned to start on his part of the commentary, Günther Frei – by then retired and living near Zurich – fell seriously ill and had to abandon the project.

Fortunately, on the recommendation of its member Norbert Schappacher, the Euler Committee was able to find a highly qualified successor who was willing to take over at short notice in January 2009. Franz Lemmermeyer, who had taken his doctoral degree in algebraic number theory with Peter Roquette in Heidelberg, had returned to Germany after some years as a visiting Associate Professor at Bilkent University, Ankara, and now teaches mathematics at a girls' high school in Ellwangen (Württemberg). He has widely published on number theory and its history; his book on the development of reciprocity laws from Euler to Eisenstein has become the standard reference on the subject.

Since Lemmermeyer is working full-time in a place where access to historical sources and older secondary literature is more limited than at the Basel University Library, some tasks had to be redistributed. In the present volume, Sections 1, 2.5–2.8 and 3 of the Introduction and most notes on historical and biographical issues have been outlined by Mattmüller; Sections 2.1–2.4 of the Introduction – its central part with respect to its scientific interest – and almost all notes on the mathematical content of the correspondence are due to Lemmermeyer. The index part has mainly been compiled by Mattmüller except for the bibliography, to which both editors have contributed. In any case, after several cycles of mutual review and revision, both editors take full responsibility for the final version of the volume.

The publication schedule had to be revised several times – partly due to the circumstances just described, partly to other tasks which claimed much of Mattmüller's time: establishing a working environment and inventory at the Basel Euler Archive, organising the manifold public activities for the tercentenary of Euler's birth at Basel, planning and effecting the transfer of the Archive to its successor, the Bernoulli-Euler-Zentrum established at the Basel University Library

in 2012. It is always hard to navigate between the opposite dangers of indefinite delay and premature, careless publication: the editors express the hope that an acceptable balance has by now been reached.

If this is indeed true, this is due – here as ever – to numerous colleagues and fellow researchers who have helped with the preparation of the present edition. Some of them should specially be pointed out, since their contribution has been crucial and the result would be significantly worse without them:

– Siegfried Bodenmann (Basel), who defined the TEX framework for the typographical design of this volume and for the setup of the multiple indices, patiently coping with the idiosyncratic specifications of the Euler Edition (and the maverick preferences of one of the editors),

– Gisela Kleinert (Halle/Saale), who inspected the transcriptions of the source texts several times with a hawk's eyes and unrelenting insistence on the consistent application of explicit rules for dealing with the many points that have to be decided one way or the other in such a long-range endeavour,

– Uta Monecke (Halle/Saale), who meticulously examined the various indices for consistency and uniformity and indefatigably helped with checking the many cross-references,

– Jordan Bell (Toronto), Fred Rickey (Cornwall, NY) and Johan Stén (Espoo, Finland), who read through the entire volume in order to help with improving the editors' sometimes stiffly accentuated English, incidentally detecting many other inaccuracies and deficiencies.

In addition to these volunteers who helped with the overall quality control of the project, the editors wish to thank the many colleagues and correspondents who have contributed individual references and corrections: Ronald S. Calinger, "engelbrekt" on MathOverflow, Lutz Felbick, Emil A. Fellmann (†), Walter Gautschi, Damaris Gehr, Sulamith Gehr, Peter Hoffmann, Vanja Hug, Gleb K. Mikhaïlov, Antonio Moretto, Fritz Nagel, Hanns-Peter Neumann, Glenn Roe, Norbert Schappacher, Thomas Steiner, Rüdiger Thiele, Andreas Verdun and Benno Zimmermann. Thanks are also due to Stephan Ammann, Heinz-Dieter Ecker, Anna Mätzener, the staff of the publishing house, Birkhäuser / Springer AG Basel, and that of the Basel University Library.

On behalf of the Editorial Commitee of Series IVA of the Euler Edition, I take this opportunity to express my sincere appreciation and gratitude to Franz Lemmermeyer and Martin Mattmüller, the editors of the present volume, for their meticulous and painstaking work. Deciphering illegible words and passages, dating undated letters, and identifying unknown persons were not the only challenges that had to be mastered. With considerable dexterity and expertness, they penetrated into the subtleties of a formalism that is often inaccessible even to those modern readers who have a solid mathematical background, and in the introduction and in hundreds of footnotes, they open the path to an intellectual world of the past that

is in many respects the foundation of present mathematics, science and technology. Thanks to their endurance and fruitful cooperation, this new edition of the Euler-Goldbach correspondence provides deep insights into the thought of two leading scholars of the Age of Enlightenment.

Andreas Kleinert
General Editor
Opera Omnia, Series IVA

LEONHARDI EULERI
OPERA OMNIA

sub auspiciis
ACADEMIAE SCIENTIARUM NATURALIUM
HELVETICAE

Edenda curaverunt

Vanja Hug, Andreas Kleinert, Martin Mattmüller, Gleb K. Mikhaïlov,
Fritz Nagel, Norbert Schappacher, Thomas Steiner

Series quarta A
COMMERCIUM EPISTOLICUM

Volumen quartum – Pars secunda

Impensis

FUNDATIONIS NATIONALIS CONFOEDERATIONIS HELVETICAE

et

ACADEMIAE SCIENTIARUM NATURALIUM HELVETICAE

Venditioni exponunt

SPRINGER BASILEAE
MMXV

LEONHARDI EULERI COMMERCIUM EPISTOLICUM

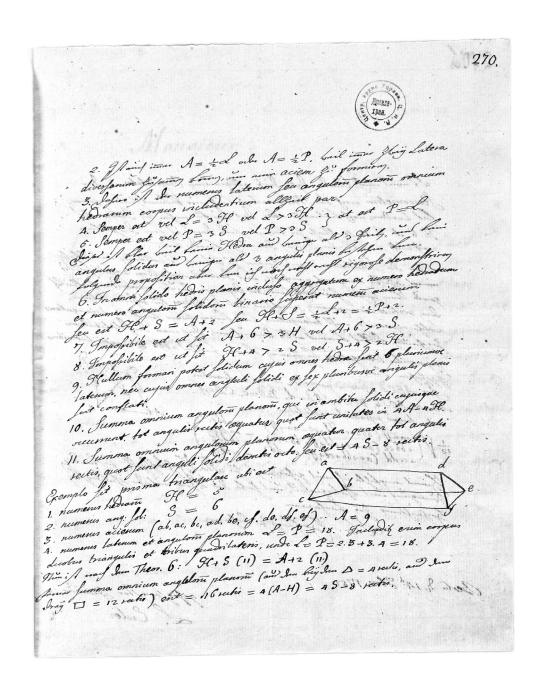CUM CHRISTIANO GOLDBACH

PARS II

Ediderunt
Franz Lemmermeyer et Martin Mattmüller

CORRESPONDENCE OF LEONHARD EULER

WITH CHRISTIAN GOLDBACH

PART II

edited by
Franz Lemmermeyer and Martin Mattmüller

SPRINGER BASEL

2015

2. $A = \frac{1}{2}L$ oder $A = \frac{1}{2}P$ ...

3. ... numerus laterum seu angulorum planorum omnium hedrarum corpus includentium allzeit par.

4. Semper est vel $L = 3H$ vel $L > 3H$ ... est et $P = L$

5. Semper est vel $P = 3S$ vel $P > 3S$ ... Hedra ... 3 angulis planis ... angulus solidus ... 3 angulis planis ...

6. ... solido hedris planis incluso aggregatum ex numero hedrarum et numero angulorum solidorum binario superat numerum acierum.
seu est $H + S = A + 2$ seu $H + S = \frac{1}{2}L + 2 = \frac{1}{2}P + 2$.

7. Impossibile est ut sit $A + 6 > 3H$ vel $A + 6 > 3S$

8. Impossibile est ut sit $H + 4 > 2S$ vel $S + 4 > 2H$

9. Nullum formari potest solidum cujus omnes hedra sint 6 plurium laterum, nec cujus omnes anguli solidi ex sex pluribusve angulis planis sint conflati.

10. Summa omnium angulorum planorum, qui in ambitu solidi cujusque occurrunt, tot angulis rectis aequatur quot sunt unitates in $4A - 4H$.

11. Summa omnium angulorum planorum aequatur quater tot anguli rectis, quot sunt anguli solidi dentis octo, seu est $= 4S - 8$ rectis.

Exemplo sit prisma triangulare ubi est

1. numerus hedrarum $H = 5$
2. numerus ang. sol. $S = 6$
3. numerus acierum $(ab, ac, bc, ad, be, cf, de, df, ef)$ $A = 9$
4. numerus laterum et angulorum planorum $L = P = 18$. Includit enim corpus duobus triangulis et tribus quadrilateris, unde $L = P = 2.3 + 3.4 = 18$.

Erit nach dem Theor. 6: $H + S$ (11) $= A + 2$ (11) ...
Summa omnium angulorum planorum ... $\triangle = 4$ rectis, ...
$\square = 12$ rectis ... erit $= 116$ rectis $= 4(A - H) = 4S - 8$ rectis.

Euler's letter n° 149 to Goldbach, November 14th, 1750: reproduction of the third page (RGADA, f. 181, n. 1413, č. IV, fol. 270r)

In the paragraph numbered 6, Euler's Polyhedron Formula appears for the first time; paragraph 11 gives the discrete form of the Gauss-Bonnet theorem (cf. n° 149, notes 5–8).

Editors
Franz Lemmermeyer
Jagstzell, Germany

Martin Mattmüller
Bernoulli-Euler-Zentrum
Basel, Switzerland

TABLE OF CONTENTS

[The introduction and the original text of the letters are contained in vol. IVA/4, Part I.]